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Notes on "Demand Deposits, Trading Restrictions, and Risk Sharking"
by
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Purpose: This paper extends the Diamond-Dybvig and Bryant analysis of liquidity demand in order to answer the following questions:

1. What happens if equity contracts are used?

2. What happens if trading in financial assets is allowed?

Jacklin concludes:

1. Dividend paying equity shares provide the same risk sharing opportunities as demand deposits, but without the possibility of a bank run in the DD preferences model.

2. For more general preferences, demand deposits provide more range for risk sharing than do equity shares.

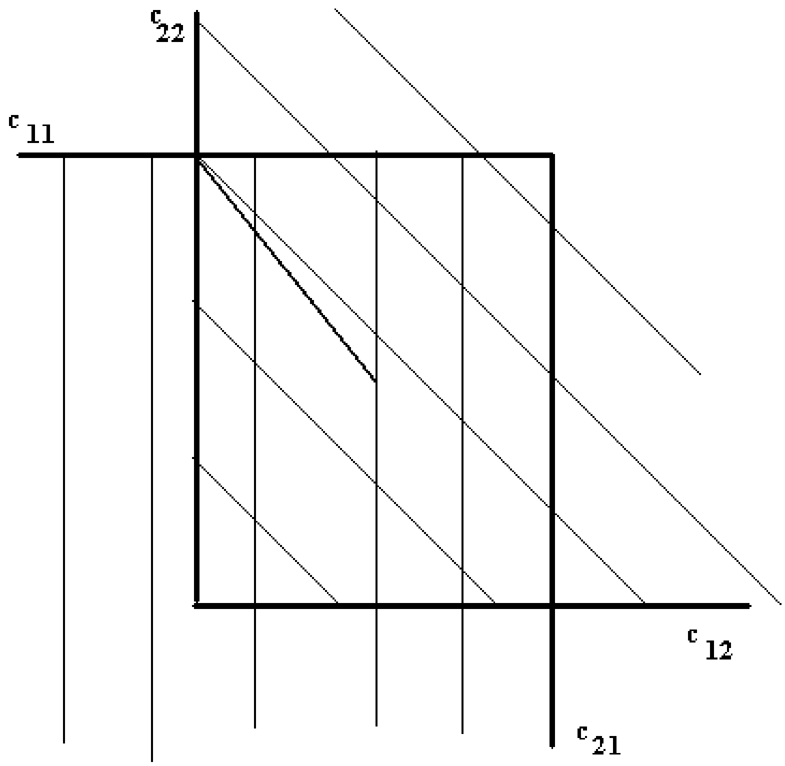
3. Opportunities to trade shares eliminate potential risk sharing by wrecking incentive compatibility.

Jacklin is uninterested in the coordination failure problem in DD; and throughout assumes that demandable debt contracts include provisions to eliminate coordination failure.

The model is in most respects identical to Diamond-Dybvig. There are three periods, 0, 1 and 2, a production technology and a storage technology. Both technologies are constant returns to scale. The storage technology is available to all participants and gives one unit of output in a later period for one unit input in an earlier period. Storage is private. The production technology gives one unit of output in period 1 for one unit of input in period 0, if production is interrupted in period 1; otherwise it gives R units of output in period 2, where R > 1.

Step 1: Initially Jacklin uses preferences identical to those in DD. However, he considers a firm which offers investors equity stakes. Essentially the firm chooses an initial investment size C, and a dividend D to be paid in the first period. Whatever is left in output is provided to shareholders in period 2. (Jacklin has the shareholders in period 0 choose the level of dividend; however since they are identical at period 0, only learning their preference shocks in period 1, they are unanimous in their preference of dividend level, and the firm would choose the same level in attempting to find a profit maximizing contract. Also note that the initial owner of the firm receives no profit—that is, the technology is freely available for anyone to duplicate.)
Thus the firm uses resources of –C in period 0, pays out D in period 1 and R(C – D) in period 2.
Assume that in period 1, individuals can trade dividends (period 1 consumption) for ex dividend shares (which yield R(C – D) of period 2 consumption per share).
Early consumers have no use for ex dividend shares and so supply them inelastically. Late consumers are indifferent between period 1 and period 2 consumption and so are willing to pay up to R(C – D) units of period 1 consumption in return for an ex dividend share. (That is, at a lower price, they will spend all their wealth on ex dividend shares). .

This arrangement achieves the optimum outcome in the planners problem, provided everybody buys the firm (C = 1) and the level of dividend D is set equal to tr_1, where r1 is the DD optimal consumption by early consumers.



Price of ex dividend shares = $r_1(1-t)$

How do we know? Start by pricing date 2 consumption relative to date 1 consumption: Everybody owns claim to t r_1 units of date 1 consumption, and to $R(1-t\, r_1)$ units of date 2 consumption. Let price be P for date 2 consumption.

Demand: If $P > 1$, late consumers will not demand date 2 consumption at all; they will simply store date 1 consumption and turn it into date 2 consumption. If $P < 1$, then they are willing to turn all their date 1 consumption into date 2 consumption. They own $(1 – t)\, t\, r_1$ units of date 1 consumption and therefore will buy $(1 – t)\, t\, r_1 /P$ units of date 2 consumption.

Supply: Since the supply is inelastic, a total of $tR(1-t\, r_1)$ sold by early consumers.

So market clearing price $P = (1 – t)\, r_1 / R(1 – t\, r_1)$, which is just total period 1 consumption divided by total period 2 consumption. This is less than 1 by the DD restriction on parameters. Thus the price of an ex dividend share is as claimed.

Note that at this price, each type consumes as required under the optimum in the planner's problem. Thus the equity arrangement in this case achieves the same outcome as the demandable debt arrangement. Jacklin says this result only applies if the shareholders can only invest in shares, but that on the other hand, as will be in step 3, the DD result only applies if the agents can only invest in demandable debt.

(Note that the constrained optimality means that all shareholders agree on this dividend policy ex ante, and the firm commits to it before they find out extra information).

Step 2: Smooth preferences. Now we generalize the preferences as follows:

$t\,U(c_{11}, c_{21}) + (1\text{-}t)\,V(c_{12}, c_{22})$.

We want it that U is impatient and V is patient; this means that

U2/U1 < V2/V1, where U1 is the partial derivative of U with respect to its first argument, etc. so that V finds period 2 consumption relatively more valuable. (Note that the same condition holds for DD preferences).

Jacklin describes a "demand deposit" as follows: It is a contract which specifies two pairs of consumption (x_1, x_2), (y_1, y_2) with the understanding that impatient types get the x pair of consumptions and patient types get the y pair. No trading of consumption or deposits is permitted.

(If sounds weird then think of a demand deposit as direct investment in two period technology + a more normal demand deposit – put minimum amount to be consumed in period 2 into direct investment, then put remnant into demand deposits. Early consumers withdraw all; late consumers strike a balance.)

The efficient allocation solves the following problem:

Max $\{c_{ij}\}$ $\quad tU + (1\text{-}t)V$

St $\quad t\,(c_{11}+c_{21}/R) + (1\text{-}\,t)(c_{12}+c_{22}/R) = 1$

(NB, if it were an individual's optimization problem, there would be two restrictions, namely $c_{11}+c_{21}/R = 1$ and $c_{12} + c_{22}/R = 1$. But insurance allows cross subsidization).

For this problem, the first order conditions are as follows:

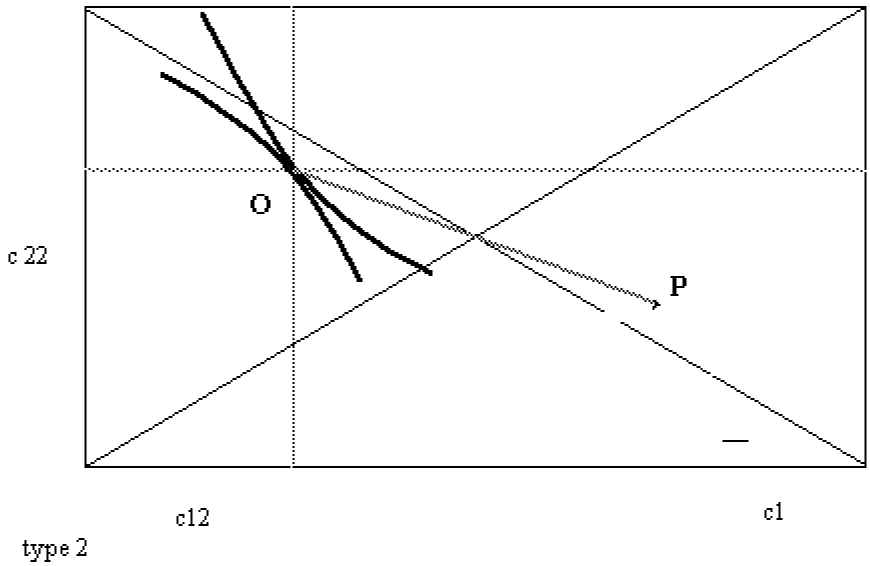
U1 = V1, U2 = V2, U1/U2 = R (plus the constraint).

For simplicity assume no individual storage. Let c_{11}^*, etc. denote the solutions to this optimization. Then given the optimal solution, there is a total amount of period 1 and period 2 consumption produced. Let's draw an edgeworth box using those totals (in fact, it is a little tricky to draw the box if the populations of early and late consumers are not the same size; for simplicity let's assume there are equal numbers of the two). The first order conditions specify that the optimal allocation is a tangency of the indifference curves, where the common slope is R.

We need to ask two questions about the optimal allocation.

1. Is it envy free? That is, does the U type prefer his allocation to the V type's allocation and vice versa? Graphically that requires that the point which is the reflection (point P in the diagram) of the allocation through equal endowment point (the center of the Edgeworth box) be less preferred by both agents to point O. If and only if O is envy free can O be supported by a demand deposit arrangement where demand deposits are not tradeable.

2. Is the present value of the two consumption patterns (at equilibrium prices) identical? (In other words, is there no cross subsidization?) Graphically, does the tangency to the two curves pass through the equal endowment point? If and only if this is the case can O be supported by an equity arrangement. This is a stricter requirement than requirement 1. (Clearly the trading outcome with equity shares is a competitive equilibrium starting from individuals holding equal endowments. Such an outcome is envy free).



In general, demandable debt contracts can give extra insurance because of the restriction to just two consumption patterns. This fact is missed in the DD preferences, since the outcome that results is always achievable as an equity contract as well. But demandable debt can achieve any individually incentive compatible (envy free) allocation. So even if the first best allocation described above is not incentive compatible, a demand deposit can achieve the second best allocation, that is, the best risk sharing allocation among those which are incentive compatible:

Max $\{c_{ij}\}$ $\quad tU + (1-t)V$

St $\quad t\,(c_{11}+c_{12}/R) + (1-t)(c_{12}+c_{22}/R) = 1$

$U(c_{11},c_{21}) \geq U(c_{12}, c_{22})$

$V(c_{12}, c_{22}) \geq V(c_{11},c_{21})$

Step 3: What are the consequences of making demand deposits tradeable?

With DD preferences, the two types do not have equal marginal rates of substitution at the optimal allocation. An individual who were to invest in the production technology directly could have a return of R in period 2 if he wanted it—this is more than he would have under a demandable debt arrangement in which late consumers subsidize early consumers. On the other hand, if he were an early consumer, he could trade his R units of late consumption at the rate of exchange of early for late consumption and make more than he could make if he were given his allocation in demandable debt.

With smooth preferences the optimal allocation will equalize marginal rates of substitution. In this case a different analysis is necessary. If there is retrading, then it must be the case that the tangency goes through the equal endowment point. For the possibility of retrading restricts us to core allocations, and in a world with continuum of agents of finite types the core reduces to the set of competitive equilibria. The demand deposit gives an agent one of two initial allocations at his discretion. He will choose whichever of the two allocations gives him the higher income, and trade from there—but then everybody will start trading with the same income, just as they would have in the case of equity trading.