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Notes on “Bank Runs, Deposit Insurance, and Liquidity”

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Summary

The models in the paper show the usefulness of bank deposit contracts as a way of providing liquidity to agents with uncertain preferences. The model also provides a way of interpreting bank runs as an inferior Nash equilibrium generated by the deposit contract. The paper also examines modifications to the demand deposit contract that improve the allocation by eliminating the inferior equilibrium.

Note: Similar results were also established by Bryant, J. (1980): “A Model of Reserves, Bank Runs, and Deposit Insurance”. *Journal of Banking and Finance 4*, pp. 335-44.

Model

There are three periods, numbered 0, 1, and 2. There is a single good in each period (input in period 0, output in periods 1 and 2).

Production: The production process has constant returns to scale. Input is taken at date 0. The production process allows the producer the option to interrupt the process at date 1, and receive a unit of date 1 output for each unit of date 0 input diverted from the production process. The remaining input (if any) continues into date 2 producing date 2 output at the rate of R units per unit of undiverted input.

Endowment: Each agent is endowed with one unit of date 0 input.

Preferences: Consumers are either type 1 or type 2. Type 1consumers (“early diers” place no value on date 2 consumption. Type 2 consumers (“late diers” place equal value on date 1 and date 2 consumption—equivalently, they value only date 2 consumption, but have a private storage technology enabling them to turn any date one consumption received into date 2 consumption at a rate of 1 for 1). As of date 0, each consumer has prior beliefs regarding his type: all individuals believe their probability of being a type 1 is t. Expected utility as of time zero is

$$t\, u(c^1{}_1) + (1 - t)\, \rho\, u(c^2{}_1 + c^2{}_2)$$

where u is concave, and $c^i{}_k$ is the consumption by the individual in period k if he is of type i.

Parametric restrictions: Assume $1 \geq \rho > R^{-1}$ and assume the coefficient of relative risk aversion is everywhere greater than one:

$$-c\, u''(c) / u'(c) > 1$$

Information: The agent's type is private information, learned by the agent at time 1. In the initial portion of the model t is known at time 0. In an extension t is stochastic. Conditional on the realization of t, all agents' realizations of their own types are independent draws.

Spot Market: In a spot market (one without insurance contracts), the outcome is autarchic. All agents are identical at time 0 and so have no gains from trade. All time zero input is put into the technology. Once agents realize their own type, there is still no gain from trading. Instead, type 1 agents withdraw from the technology and each consumes 1. Type 2 agents continue with the technology and each consumes R at time 2. Thus $c^1_1 = 1$, $c^2_1 = 0$, and $c^2_2 = R$, so that utility under autarchy is

$$t\, u(1) + (1 - t)\, \rho\, u(R)$$

First Best Insurance Contract: Suppose there are large numbers of consumers such that aggregate uncertainty can be ignored: precisely a fraction t turn out to be type 1s and the rest type 2s. Furthermore suppose the realization of an agent's type were public information at time 1. Then a mutual insurance contract could be established at time 0. The optimal such contract would maximize the representative agent's ex ante utility

$$t\, u(c^1_1) + (1 - t)\, \rho\, u(c^2_2)$$

subject to a feasibility restriction:

$$t\, c^1_1 + (1 - t)\, c^2_2/R \leq 1.$$

(Note that we have dropped reference to c^2_1 since it would be suboptimal to have type 2 consumers take any period 1 output). The conditions for an optimum are

$$u'(c^*_1) / u'(c^*_2) = \rho R$$

$$t\, c^*_1 + (1 - t)\, c^*_2/R = 1.$$

(Assuming the Inada conditions on the function u guarantees that the optimum is an interior point.)

Since $\rho R > 1$ we know that $c^*_1 < c^*_2$. That is, under optimal risk sharing, late diers consume more than early diers.

Since the coefficient of relative risk aversion is greater than 1 we conclude that

$$1 < c^*_1 < c^*_2 < R.$$

That is, optimal risk sharing gives early diers more consumption than they would have under autarchy.

(To see this, note that saying that the coefficient of relative risk aversion is greater than 1 is the same as saying that the expression $c\, u'(c)$ is a decreasing function. Thus $u'(1) > R\, u'(R)$ and a fortiori

$$u'(1) > \rho R\, u'(R)$$

but at first best insurance

$$u'(c^*_1) = \rho R\, u'(c^*_2).$$

Thus to move from the consumption pattern in autarchy to the consumption pattern in first best insurance (both of which are feasible) requires transferring consumption from late diers to early diers.)

<u>Insurance with Private Information:</u> Now return to the situation in which consumers' types are private information. Is it incentive compatible for consumers to reveal their types? Yes. A type 1 consumer who pretended to be a type 2 consumer would get type 2 consumption, which is worthless to him. A type 2 consumer who pretended to be a type 1 consumer would get period 1 consumption, which is worth the same to him as period 2 consumption, but since under complete insurance period 1 consumption is lower than period 2 consumption, this is not a switch he would choose to make.

Thus, if we put a contract together in which people announce their type, it is in my interest as an individual agent to tell the truth about my type, *assuming all the others do*. It is not possible to a contract which promises *every* consumer who claims to be an early dier the amount c^*_1 *regardless* of what others claim to be, because $c^*_1 > 1$. Thus giving everybody that amount, for example, would be infeasible.

To talk about mechanisms that implement the first best outcome, we need to describe more carefully what a mechanism will give to participants in every possible situation, and to make sure it is always feasible.

<u>Sequential Service Constraint:</u> In this environment, each agent has only one of two announcements to make: either he is an early dier or a late dier. Thus it is perfectly natural to translate this announcement into an action of "withdrawing funds." Early diers announce their status by attempting to withdraw funds from the bank at date 1. In order to focus on mechanisms which "look like" banks, Diamond and Dybvig confine their attention to mechanisms which satisfy the "sequential service constraint." By this they mean that a mechanism lines up all early withdrawers (randomly) and then pays each according to his place in line, independently of the number of individuals who are further back in the line. The simple demand deposit contract with fixed early payment r_1 pays a fixed amount r_1 to any individual who asks for an early withdrawal, as long as there is enough in the bank to do so. Early withdrawers beyond that limit get zero. Participants who do not withdraw early get a pro rata share of the bank's remaining assets in period 2. Note that this is a complete specification of a mechanism, since it states a feasible allocation for every possible combination of announcements.

If we set $r_1 = c^*_1$ then it is incentive compatible for agents to withdraw funds early if and only if they are early diers, provided others behave in the same way. That is, such behavior is a Nash equilibrium. There is, however, a second Nash equilibrium in the mechanism. In this second Nash equilibrium, all agents attempt to withdraw early. Given that others are doing so, there will be nothing left for late withdrawers, and it is therefore in each agent's interest to attempt to do so as well. Diamond and Dybvig interpret this second equilibrium as a "bank run." They argue that the arrangement is inherently unstable in that either of the equilibria is possible, and irrelevant variables ("sunspots") could trigger a change from one equilibrium to the other.

Since this mechanism has the associated instability, it is natural to look for modifications that eliminate the undesirable equilibrium. The key is to make sure that there is enough left in the bank to make it desirable for late diers not to withdraw early. If we ensure that no more than f agents are permitted to withdraw early, then the remaining agents will each be able to receive at least the amount

$$R(1-f\,r_1)/(1-f).$$

Thus if this amount is greater than r_1, then no agent has an incentive to withdraw prematurely. Setting f to be somewhere in the interval

$$[t, (R-c^*_1)/(Rc^*_1-c^*_1)]$$

allows us to achieve the first best as the only equilibrium, since truth telling is now a *dominant* strategy. In other words, the mechanism works the same as before, but only the first f people who ask for early withdrawal are granted it; the rest are told to wait until the second period. Diamond and Dybvig identify this modification with "suspension of convertibility." Note that this modification continues to satisfy the sequential service constraint.

Stochastic Withdrawals: Suppose now that the number of early diers is a random variable. The conditions for optimal risk sharing are unaffected, except that the variable t in the conditions is now a random variable (non diversifiable risk). Individual incentive compatibility would continue to hold, and it would be in general possible to develop an insurance contract that implements the first best. However it is not possible to develop such a contract while also satisfying the sequential service constraint. This is because the amounts that are paid to each early dier depend on the total number of early diers: we need to know how many people are in the line before we start handing out early withdrawals.

Government Deposit Insurance: Allowing the government to impose first period taxes and use the proceeds to benefit late withdrawers enables the implementation of the first best even in the case that the bank's scheme is restricted to contracts satisfying the sequential service constraint. The key features to make this work are that 1) the government is by assumption permitted to vary its taxes based on the observed level of withdrawals 2) the government is assumed able to deposit the proceeds into the bank (thereby allowing the bank to continue to get the return R to the second period on all funds)

Evaluation

The paper provides a convincing justification for intermediaries as providers of liquidity. It shows that a simple demandable debt contract can provide such liquidity and it emphasizes that terms of a demandable debt contract may leave the bank unable to honor the contract in the face a run by depositors. However, the "suspension of convertibility" of the contract does not look like historical suspension of convertibility. The sequential service constraint is unmotivated: the paper shows there will be real consequences from restricting a contract not to depend on aggregate preferences, but it provides no rationale for such a restriction. The main point of the "deposit insurance" is to demonstrate that there are ways around the sequential service constraint. In fact the government's powers in deposit insurance in the model are so extensive that they could make a variety of arrangements into optimal arrangements.